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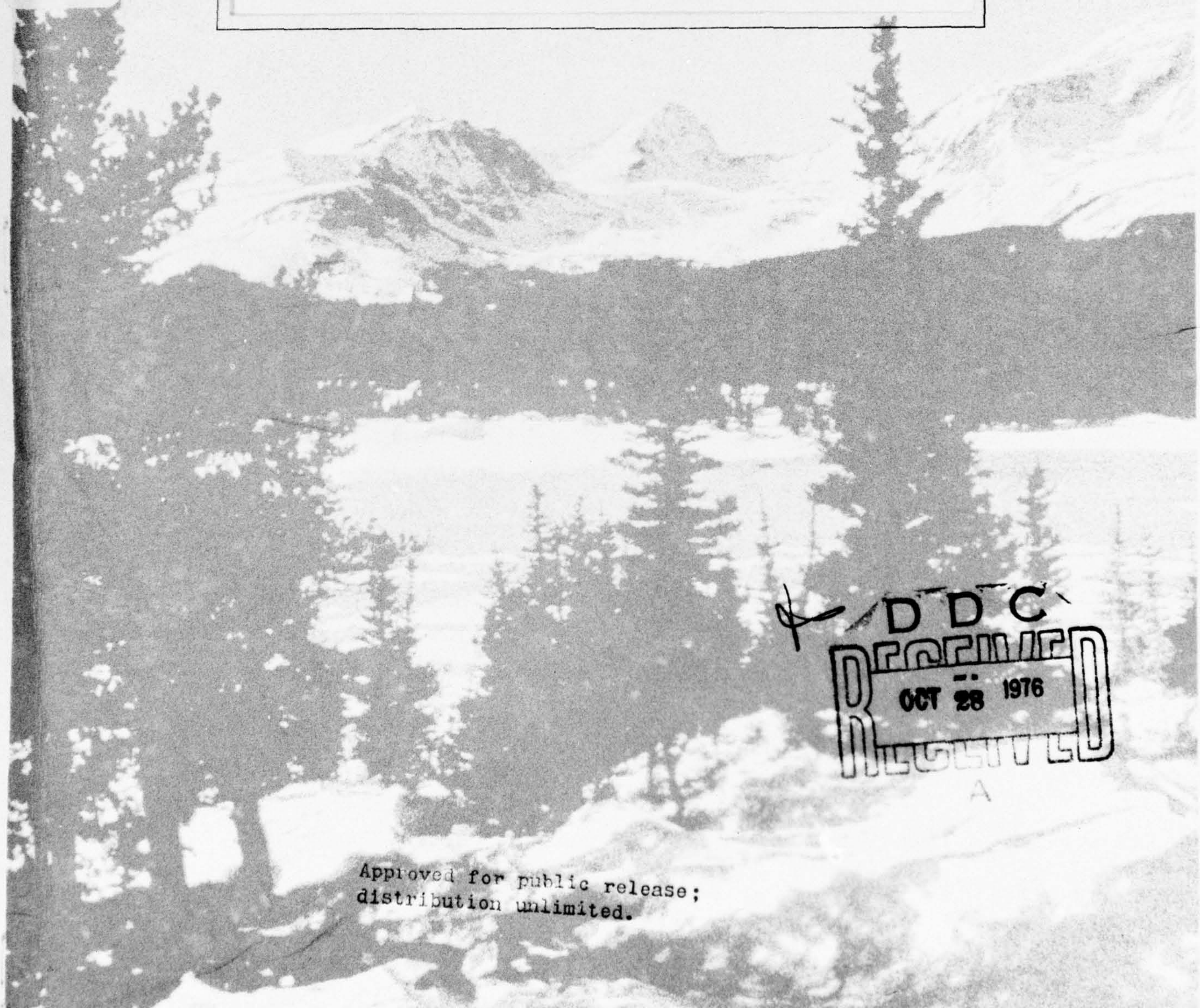

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LANGMUIR SHOCK WAVES

Martin V. Goldman and Dwight R. Nicholson

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Langmuir Shock Waves

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ABSTRACT

Intense Langmuir waves in a uniform, one-dimensional plasma are described by a nonlinear Schrödinger equation. When the effect of linear Landau damping is included, we find steady state shock wave solutions. For an infinite plasma, frequency determines shock amplitude and (subsonic) speed. For a semi-infinite plasma, the shock is stationary; its frequency determines its amplitude, and the energy flux through the boundary determines the spatial extent of the field.

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I. INTRODUCTION

It is well known that, under certain conditions, intense Langmuir waves characterized by electric field $[\tilde{E}(x,t)\exp(-i\omega_{po}t) + c.c.]$ and the associated ion density perturbation $n_i(x,t)$ are described in time and in one spatial dimension by the dimensionless equations

$$i\,\frac{\partial \tilde{E}}{\partial t} + \frac{\partial^2 \tilde{E}}{\partial x^2} - n_i\tilde{E} + iL(\tilde{E}) \;=\; 0 \tag{1}$$

and

$$\frac{3}{4}\,\frac{m_i}{m_e}\,\frac{\partial^2 n_i}{\partial t^2} - \frac{\partial^2 n_i}{\partial x^2} \;=\; \frac{\partial^2 |\tilde{E}|^2}{\partial x^2} \quad , \tag{2}$$

where we have included a linear damping operator L(E) in (1) and m_i and m_e are the ion mass and electron mass respectively. Physical variables are obtained from the dimensionless variables in (1) and (2) as follows: distance is $3^{1/2}\lambda_e x$, where $\lambda_e = (T_e/m_e)^{1/2}$ is the electron Debye length; time is equal to $2t/\omega_{po}$, where ω_{po} is the unperturbed background electron plasma frequency; the electric field is equal to $\tilde{E}(x,t)(4m_eT_e)^{1/2}\omega_{po}/e$, where e is the absolute value of the electron charge; ion density perturbation is n_in_o, where n_o is the unperturbed ion density; the ion temperature $T_i = 0$ and $\tilde{E}(x,t)$ and $n_i(x,t)$ vary on a time scale long compared to ω_{po}^{-1}.

An extensive bibliography of work on these equations can

be found in Nicholson and Goldman,[1] who study the temporal damping of soliton-like initial conditions. Appendix A of the present paper provides a list of the assumptions which enter the derivation of (1) and (2).

Suppose there is a frame (where the ion disturbance is stationary) moving with velocity V with respect to the laboratory. If $z = x-Vt$, we can solve (2) for n_i; defining $E(x,t) = \tilde{E}(x,t)\left[1 - V^2(3m_i/4m_e)\right]^{-1/2}$, Eq. (1) becomes

$$i \frac{\partial E}{\partial t} - iV \frac{\partial E}{\partial z} + \frac{\partial^2 E}{\partial z^2} + |E|^2 E + iL(E) = 0 . \qquad (3)$$

This result is valid for speeds $|V| << (4m_e/3m_i)^{1/2}$ or in physical units, speed less than sound speed $c_s \equiv (T_e/m_i)^{1/2}$.

Using Eq. (3), Nicholson and Goldman[1] studied the damping of a solitary wave initial condition. This work is analogous to the work of Ott and Sudan[2,3] on the Korteweg-deVries equation.

In the present paper, we discuss solutions of (3) which exhibit shock-like behavior; that is, the field at $x \to -\infty$, $|E(x = -\infty, t)|$ has a different value than it does at some other point in space, either $x = +\infty$ for the case of infinite plasma, or $x = 0$ for the case of a semi-infinite plasma.

Section II presents an analytic approach to Eq. (3). In Section III we discuss numerical solutions to (3) with an infinite medium and L(E) representing linear Landau

damping; here we find that the shock amplitude implies shock speed and vice versa. Section IV discusses steady state shock solutions with zero speed for a semi-infinite medium, with a constant energy flux injected at the boundary. Conclusions and related work are discussed in Section V.

II. ANALYTIC CONSIDERATIONS

We look for a solution of (3) of the form

$$E(z,t) = \rho^{1/2}(z)\, e^{i\sigma(z)+i\Omega t} , \tag{4}$$

where the amplitude $\rho^{1/2}(z)$ is independent of time. Inserting (4) into (3) and separating real and imaginary parts we have

$$\frac{1}{2}\rho'^2 + \rho^3 + 2\rho^2\left[\frac{V^2}{4} - \Omega\right]$$

$$= 2\rho \int_{-\infty}^{z} dz'\rho'(z')\left(\tilde{I}^2(z) - \frac{R(z')}{\rho^{1/2}(z')}\right) \tag{5}$$

and

$$\sigma' = (V/2) + \tilde{I} , \tag{6}$$

where $(\ )' \equiv \partial(\ )/\partial z$ and

$$\tilde{I}(z) \equiv -\frac{1}{\rho(z)} \int_{-\infty}^{z} dz'\rho^{1/2}(z')I(z') \tag{7}$$

and

$$\begin{bmatrix} R(z) \\ \\ I(z) \end{bmatrix} = \begin{pmatrix} \mathrm{Re} \\ \\ \mathrm{Im} \end{pmatrix} \left[ie^{-i\sigma} L(\rho^{1/2} e^{i\sigma}) \right] . \tag{8}$$

(In Appendix B we discuss Eq. (5) rewritten in terms of $\rho^{1/2}$ instead of ρ; this is more instructive for some purposes.) Equation (5) is analogous to a particle energy equation, of the form

$$\frac{1}{2}\,\rho'^{\,2} + \Phi(\rho) = W(\rho,z) \xrightarrow[\text{No Damping}]{} 0 \quad , \tag{9}$$

where $(1/2)\rho'^{\,2}$ is the "kinetic energy", $\Phi(\rho)$ is the pseudopotential, and $W(\rho,z)$ is the "total energy"; and where the amplitude $\rho(z)$ plays the role of "particle velocity" and the position z plays the role of "time." The "total energy" $W(\rho,z)$ goes to zero as $z \to -\infty$, and is identically zero in the absence of damping. The pseudopotential $\Phi(\rho) = \rho^3 + 2\rho^2[(V^2/4) - \Omega]$.

For zero damping, we obtain the soliton solution as shown in Fig. (1a). A "particle" in the potential well takes an infinite "time" z to roll from the initial "position" $\rho = 0$ to the reflection "position," $\rho = \rho_{MAX}$, and an infinite amount of "time" z to roll back again. In this undamped case, Eq. (6) is trivially solved to obtain $\sigma(z) = V_z/2$, and the pseudopotential equation (5)

can be analytically solved for $\rho(z)$. After transforming back to the laboratory frame $x = z+Vt$, we have the well-known class of single soliton solutions

$$E(x,t) = \left(2\Omega - \frac{V^2}{2}\right)^{1/2} \operatorname{sech}\left[\left(\Omega - \frac{V^2}{4}\right)^{1/2}(x-Vt)\right]$$

$$\exp\left(i\Omega t - \frac{iV^2 t}{2} + \frac{iV}{2}x\right) \quad , \tag{9}$$

or, in terms of $A \equiv [\Omega - (V^2/4)]^{1/2}$,

$$E(x,t) = 2^{1/2}A \operatorname{sech}[A(x-Vt)] \exp\left(iA^2 t - \frac{iV^2 t}{4} + \frac{iV}{2}x\right) \tag{10}$$

which exist for $0 < A^2 << [1 - V^2(3/4)(m_i/m_e)]$ and $|V| < (4m_e/3m_i)^{1/2}$.

With the inclusion of Landau damping in (5)-(8), it can be verified numerically, as in the following two sections, that the total energy $W(\rho,z)$ decreases slowly (not always monotonically) with "time" z as the "particle" rolls in the well, yield a shock solution as shown in Fig. (1b). In Section III we numerically obtain an infinitely extended shock; the shock speed is determined by the shock amplitude. In Section IV we find semi-infinitely extended shocks, which are stationary and have an energy flux through the boundary.

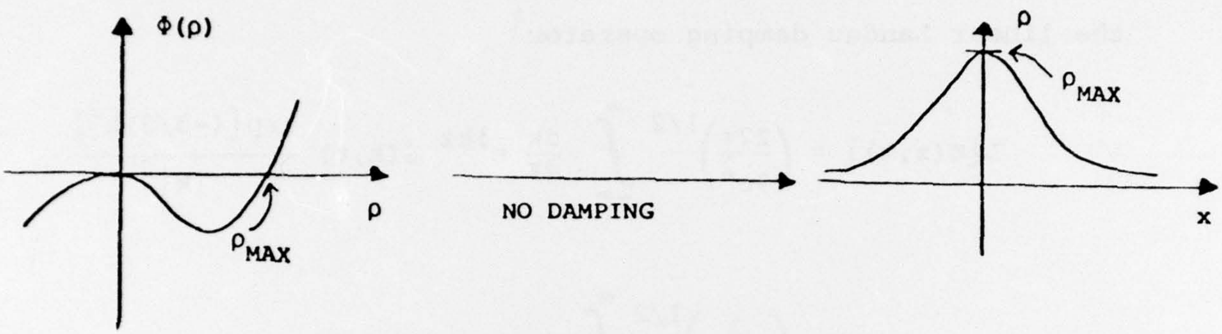

FIGURE 1a

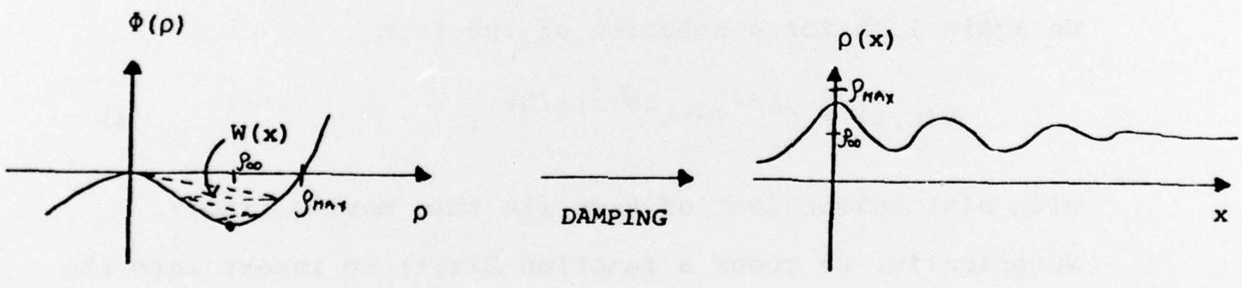

FIGURE 1b

III. INFINITE SHOCK

We obtain an infinitely extended Langmuir shock wave by treating Eq. (3) numerically with the inclusion of the linear Landau damping operator[1]

$$L[E(z,t)] = \left(\frac{27\pi}{\partial e^2}\right)^{1/2} \int_{-\infty}^{\infty} \frac{dk}{\partial\pi}\, e^{ikz}\, E(k,t)\, \frac{\exp[(-3/2)k^2]}{|k|^3}$$

$$= \left(\frac{3}{2e^3\pi}\right)^{1/2} \int_{-\infty}^{\infty} dz'(z-z')E(z',t)$$

$$\int_{0}^{\infty} dk \exp[(-3/2)k^2]\sin[k(z-z')] \ . \qquad (11)$$

We again look for a solution of the form

$$E(z,t) = \rho^{1/2}(z)e^{i\sigma(z)+i\Omega t} \ , \qquad (4)$$

with $\rho(z)$ independent of time (in this moving frame). Numerically, we guess a function $E(z,t)$ to insert into the damping operator L in (11). Specifically, we guess the soliton in Eq. (9); we then numerically integrate the (now time-independent) Eq. (3) from $z \to -\infty$, taking the soliton form of the solution from (9) as the boundary condition at large negative z. The resulting $E(z,t)$ is

then inserted into the damping operator (11), and the entire procedure is iterated until convergence is obtained. (The convergence is usually quite rapid.)

The only allowable solutions are those for which $\sigma(z) \to$ constant as $z \to +\infty$, and $\rho(z) \to$ constant as $z \to +\infty$. If this were not the case, then the Landau damping operator would be finite over an infinite region of space, yielding an infinite amount of energy dissipation per unit time and no possibility of a steady state shock solution.

The constancy of $\sigma(z)$ and $\rho(z)$ at large z implies that there can be no energy flux from large z into the region of Landau damping. To establish this fact, we use energy conservation laws: multiply (3) by E*, multiply the complex conjugate of (3) by E, subtract and integrate over all space; we find in the shock frame, using the fact that $E(z \to -\infty, t) \to 0$,

$$-V|E|^2\Big|_{z\to+\infty} + 2\mathrm{Im}\left[E^*(\partial E/\partial z)\right]\Big|_{z\to+\infty}$$

$$= -2\mathrm{Re}\int_{-\infty}^{\infty} dz' E^* L(E) \ . \tag{12}$$

The second term, assuming $\rho(z)$ constant as $z \to +\infty$, is

$$2\rho(z)\ (\partial\sigma/\partial z)\Big|_{z\to\infty} \ . \tag{13}$$

This term, representing energy flux from $z \to +\infty$, vanishes

since $\partial\sigma/\partial z|_{z\to\infty} \to 0$ by the argument above. The remaining two terms show that the energy dissipated by Landau damping (right-hand side) is exactly balanced by $-V|E|^2|_{z\to+\infty}$, which is just the rate at which electric field energy is disappearing from a fixed spatial volume of the laboratory frame because the shock is moving to the right with speed V.

Now we already know the value of $|E|^2|_{z\to+\infty} \equiv \rho_\infty$ in terms of the pseudopotential of Fig. (1b); this is just the point where $d\Phi(\rho)/d\rho \to 0$ or $\rho_\infty = (4/3)[\Omega - (V^2/4)] = (2/3)\rho_{MAX}$. Thus, (12) becomes an implicit expression for V:

$$V = \frac{3}{2[\Omega-(V^2/4)]} \operatorname{Re} \int_{-\infty}^{\infty} dz' E^*(z',t) L[E(z',t)] \; , \qquad (14)$$

in which it must be remembered that E(z,t) obtained numerically depends on the value of V used in (3). Equation (14) can also be obtained from (6) and (7) by setting $\sigma'(z\to+\infty) = 0$. In practice, we must numerically repeat this calculation with different values of V until we find the one which makes $d\sigma/dz|_{z\to+\infty} \to 0$.

The results of this procedure are shown in Fig. 2 for $\Omega = 0.3$; the value $V = 0.35$ is found numerically to make $d\sigma/dz|_{z\to+\infty} \to 0$.

The energy conservation law (12) indicates (it may be difficult to prove this rigorously) that for each shock amplitude there is a shock speed, and vice versa. For a

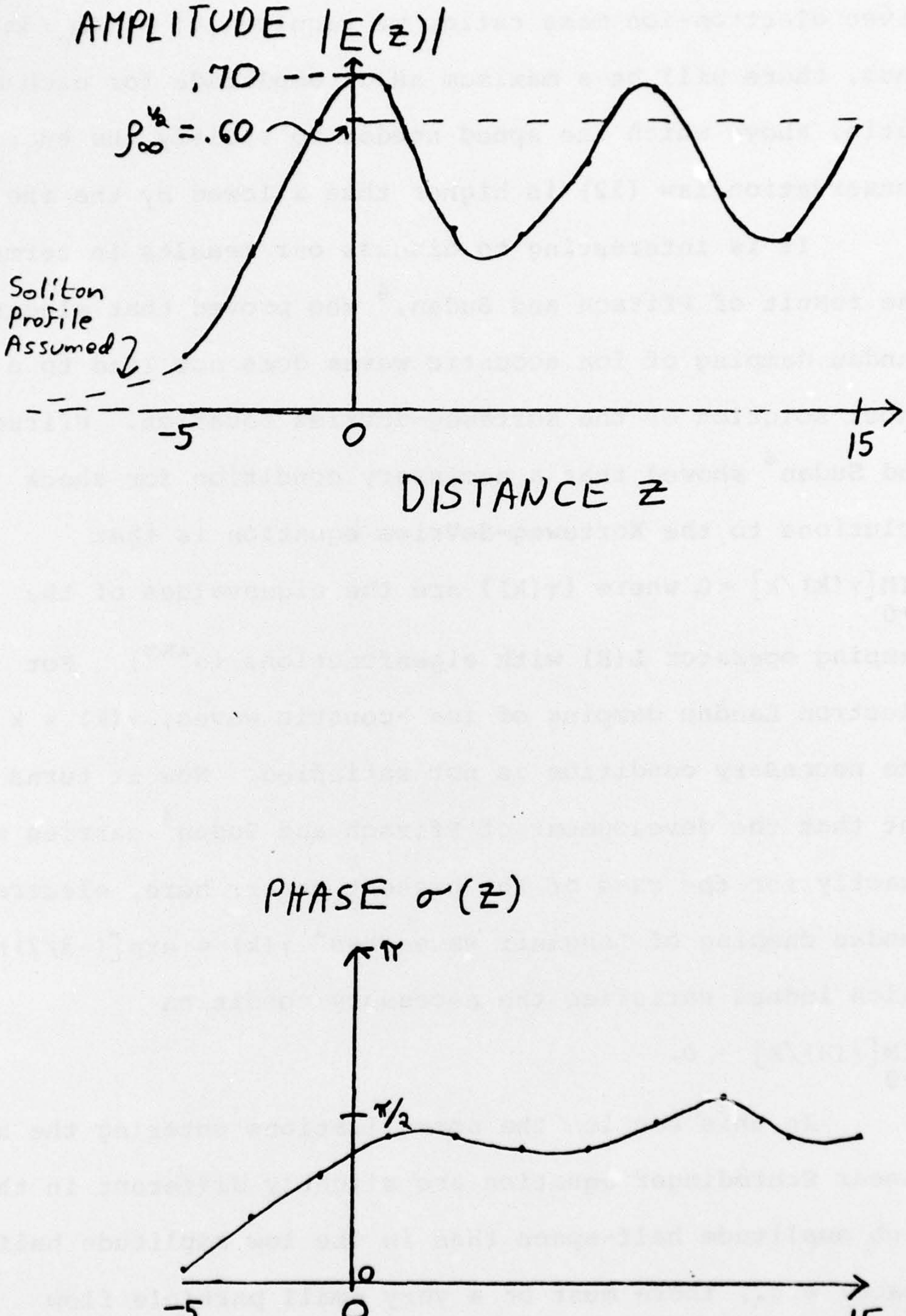
AMPLITUDE |E(z)|
.70
$\rho_\infty^{1/2} = .60$
Soliton
Profile
Assumed
-5
0
15
DISTANCE z
PHASE $\sigma(z)$
π
$\pi/2$
0
-5
0
15
DISTANCE z

FIGURE 2

given electron-ion mass ratio, we require $|V| << (4m_e/3m_i)^{1/2}$. Thus, there will be a maximum shock amplitude for each mass ratio, above which the speed needed to satisfy the energy conservation law (12) is higher than allowed by the theory.

It is interesting to discuss our results in terms of the result of Pfirsch and Sudan,[4] who proved that electron Landau damping of ion acoustic waves does not lead to a shock solution of the Korteweg-deVries equation. Pfirsch and Sudan[4] showed that a necessary condition for shock solutions to the Korteweg-deVries equation is that $\lim_{k\to 0}[\gamma(k)/k] \to 0$, where $\{\gamma(k)\}$ are the eigenvalues of the damping operator L(E) with eigenfunctions $\{e^{ikx}\}$. For electron Landau damping of ion acoustic waves, $\gamma(k) \propto k$ and the necessary condition is not satisfied. Now it turns out that the development of Pfirsch and Sudan[4] carries over exactly for the case of the present paper; here, electron Landau damping of Langmuir waves has[1] $\gamma(k) \propto \exp[(-3/2)k^2]/|k|^3$ which indeed satisfies the necessary condition $\lim_{k\to 0}[\gamma(k)/k] \to 0$.

In this section the normalizations entering the non-linear Schrödinger equation are slightly different in the high amplitude half-space than in the low amplitude half-space; e.g., there must be a very small particle flow velocity in the lab frame, high-amplitude half-space, to supply particles to the region where the shock front has

just passed. These differences in normalization are ignored, consistent with the derivation of the basic equations (Appendix A) and with previous work on nonlinear Schrödinger equation shocks.[5]

IV. SEMI-INFINITE SHOCK

Let us now consider a semi-infinite medium, $-\infty < x \leq L$. We again look for shock solutions, but here we allow an energy flux through the boundary at $x = L$, traveling to the left. We use the numerical procedure of the previous section, modifying the spatial integral in (11) so that the limits of integration are $z \to -\infty$ and $z = L$. The results are qualitatively similar to those of Fig. (2) for the infinite medium, with two differences: namely, that the right-side of the graph now represents the physical boundary at $z = L$, with boundary conditions at that point on $|E(z=L)|$, $\sigma(z=L)$, and their derivatives; and that, in contrast to the previous section, there is no requirement that $\sigma'(z \to L) \to 0$, since this derivative is indeed proportional to the energy flux.

Numerically, the frequency of the shock determines the shock amplitude, and the length of the system determines the energy flux required. Physically, we would like to make the argument in the opposite direction. That is,

given a semi-infinite system with a boundary at $z = L$, we apply an electric field at a given frequency and with the correct amplitude for that frequency; and we adjust the spatial derivative of the electric field so that there is a certain energy flux. Then a response moves into the plasma, forming a shock wave; when it has penetrated so far that the energy dissipation, integrated over $-\infty < z < L$, exactly balances the energy flux through the boundary, the shock stops moving and we have a steady state which looks qualitatively like that of Fig. (2).

If the above scenario actually occurs, it may have many implications for laser fusion, microwave heating of magnetically confined plasma, radio wave modification of the ionosphere, and many other physical situations. The question of whether the above scenario actually occurs is strongly related to whether the shock wave is stable. We have undertaken the stability analysis, which will be discussed in a separate publication. The evidence for stability from the two known limiting cases is ambiguous; the high amplitude part of the infinite shock of the previous section is known to be modulationally unstable, while the soliton shape itself is known to be stable. Thus, a careful stability analysis is required for the semi-infinite shock.

V. CONCLUSIONS AND RELATED WORK

In an infinite medium, we have found that the combination of electron Landau damping and ponderomotive force nonlinearity leads to the existence of shock-like solutions, traveling to the right (toward the high amplitude region) in order that the dissipation energy can be balanced by a disappearance of electric field energy in the system. There is a one-to-one correspondence between shock amplitude and shock speed; for a given electron-ion mass ratio there is a maximum allowable shock amplitude.

A semi-infinite shock is found, stationary in the lab frame, with energy flux balanced by dissipation.

Our work differs from that of Mima, Nishikawa, and Ikezi.[5] We treat electron nonlinearity and electron Landau damping; our shock speed is $0 \leq |V| < c_s$ in physical units. They treat electron nonlinearity, ion nonlinearity ($\underline{v} \cdot \nabla\underline{v}$), and reflected ions; their shock speed is limited to $|V| \approx c_s$ in physical units. We expect that very close to the ion sound speed, ion nonlinearities will become important in our work, and the treatment may need modification.

APPENDIX A: ASSUMPTIONS IN DERIVING THE NONLINEAR SCHRÖDINGER EQUATION

The basic equations (1) and (2) are derived from the force equation, continuity equation, and Poisson's equation under the following assumptions:

(i) All quantities can be characterized as high frequency (ℓ) or low frequency (s).

(ii) Quasineutrality: (a) The oscillatory distance of an electron in the high frequency electric field, x_{OSC}, must be much less than the low frequency scale length, L^s; $x_{OSC} << L^s$. (b) We require $\lambda_e << L$, where λ_e is the electron Debye length.

(iii) Weak turbulence: $|E^\ell|^2 << n_o T_e$ leads to $x_{OSC} << \lambda_e$ and to $\delta n_e{}^s \simeq \delta n_i << n_o$, where E^ℓ is the high frequency electric field, n_o is the background density, and $\delta n_{e,i}$ is the (electron, ion) density disturbance.

(iv) Where $V_e{}^s$ is the low frequency speed, v_i is the ion speed, and $v_e{}^\ell$ is the high frequency electron speed, $v_e{}^s \sim v_i << (m_e/m_i)^{1/2} v_e{}^\ell$.

APPENDIX B: ALTERNATE PSEUDOPOTENTIAL

In Section II the pseudopotential equation (5) uses $\rho(z)$ as a variable. If we rewrite that equation in terms of $\eta(z) \equiv \rho^{1/2}(z)$, we obtain

$$\frac{1}{2}\,\eta'^{2} + \frac{1}{4}\,\eta^{4} + \frac{1}{2}\,\eta^{2}\left(\frac{V^{2}}{4} - \Omega\right) = \tilde{W}(\eta,z) \;, \qquad \text{(A1)}$$

where

$$\tilde{W}(\eta,z) = \frac{1}{2}\int_{-\infty}^{\infty} dz'\left[\eta^{2}(z')\right]'\left[\tilde{I}^{2}(z') - \frac{R(z')}{\eta(z')}\right], \qquad \text{(A2)}$$

and $\tilde{I}$ and R are defined in (7) and (8). We have found that the right-hand side of the former pseudopotential equation (5) is not a monotonically decreasing function of z; it is interesting to speculate whether the right-hand side ($\tilde{W}(\eta,z)$) of the new pseudopotential equation (A1) is a monotonically decreasing function of z. Numerical work to answer this question is in progress.

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